VI. *An Investigation into the Periodicity of Measles Epidemics in London from 1703 to the Present Day by the Method of the Periodogram.*

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In two short papers* published during the last two years, I have given an account of some preliminary investigations into the periodicity of outbreaks of epidemic disease, by the method of the periodogram. The treatment in both cases was elementary and aimed rather at the discovery of the main periods in each instance than at the minute investigation of any one disease for a long series of years.

A complete examination of the statistics of one disease was, therefore, greatly to be desired. This was further necessary as possibly affording a means of distinguishing between rival theories of epidemics.

Such an investigation with regard to measles in the City of London has now been carried out. For this example there is quite an exceptional amount of evidence. The London Weekly Bills of Mortality preserved in the libraries of the Guildhall and the British Museum contain information which gives, among other details, the number of deaths from measles for every week from the year 1703 to the year 1838, except for an unfortunate break of 12 months in the years 1829–30. From 1838 the publications of the Registrar-General give the same information. The fact that the data refer to deaths alone and not to the cases must be regarded as a subsidiary drawback, since, on comparison, the statistics of cases and deaths where the information for both is available (*e.g.* Aberdeen, 1880–1900) show a very close correspondence. It is not, of course, in the least contended that for the eighteenth century the data are nearly as accurate as those available since the introduction of registration, but for the purpose of this investigation the relative accuracy of figures for considerable periods of time is nearly as important. Were this relative accuracy even approached, the figures of the eighteenth century would have been of extreme value. But as will be seen later, though some conclusions may be definitely arrived at, still the figures cannot unfortunately be taken as representing any uniform system of record.

Further, measles is an easy disease to diagnose, even in the absence of medical skill. Few conditions simulate it. It may, at times, be mistaken for scarlet fever or *vice versâ*. The seasonal prevalence of the two diseases, however, is different; and it has been found that in specific instances in which the accuracy of the figures

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in the Bills of Mortality has been questioned, that the deaths alleged by the critics to be due to scarlet fever have occurred at seasons of the year at which it would be unusual to have an outbreak of that disease. Thus, to take the year 1786, as to which WILLAN and SIMS state in their opinion that many deaths recorded as measles were in reality due to scarlet fever, it is found that the two epidemics ran different courses each in accordance with its own proper natural history. It is possible that some wrong diagnosis occurred, but it is probable that this had only a minor effect.

Measles is, in addition, a disease with short intervals between the epidemics. The periods are roughly two years. Consequently in a series of 210 years, more than a hundred periods are included; amply sufficient to determine if the periods are constant or vary in length from time to time. No other easily recognisable disease exists for which anything like such minute information is available, though had vaccination not been discovered the record might have been matched by that of small-pox, the only other possible rival. This investigation has been carried out for three epochs:* (1) 1703–1754, (2) 1755–1828, (3) 1838–1912.

The results of the application of the method to the statistics of the third epoch will be discussed first, as the figures referring to this epoch are much the most authoritative. Unfortunately the number of deaths cannot be directly used. Measles is a disease which causes death almost solely among children under five years of age. Between 1840 and 1880 the number of children in London under five years of age doubled itself. Thus the earlier figures have no correspondence with the latter. The correction has been made on the assumption that the population at ages under five years has remained constant. In one way this is quite satisfactory, but in another not so satisfactory. It cannot be said *a priori* that the course of two epidemics—all conditions, such as infectivity, weather, etc., being equal—would be exactly the same if the epidemics occurred among two populations, one of which was twice the size of the other, even although the numbers of the smaller population were sufficiently large to make the error due to random selection more or less negligible. There are biological factors in such cases which must be experimentally determined and not deduced merely by the consideration of what seems probable. But a correction had to be made, and as the correction does not introduce any periodic element it can hardly have the effect of introducing spurious periods.

The figures for each week have been written out in a series of periods differing by one week, beginning with 80 weeks and rising to 120 weeks. From 120 weeks to 160 weeks the interval between the periods is two weeks. From three years the investigation has been carried up to a limit of 25 years at intervals of quarter years. As the figures only relate to 75 consecutive years it has not been possible to extend the investigation further. For each of these periods the amplitudes of the first three

* To avoid confusion, the three series of periods of time will be considered throughout this paper as "epochs," and the word period reserved strictly to express the duration of time between successive epidemics.

harmonics have been calculated, and, in some instances, that of the fourth as well. This has been done by reducing the number of elements in each of the totals to 20, which seems sufficient to give the harmonic amplitudes with fair exactness. In general, the minimum numbers were selected as the point of start. This procedure certainly tends to reduce the amplitudes, but in a case selected at random in which a 60-term analysis was tested against one of 20 terms the difference found was only 2 per cent. As the method has been applied uniformly it probably does not conceal any significant fact; 20 terms certainly give better results than 12 and the use of the former number does not seriously increase the labour of computation. Though the arithmetical calculation of the amplitudes, with a few exceptions, has only been carried out for the totals of the 75 years, yet the first additions for each period were made in four groups each including the statistics of from 18 to 19 years. The results of these separate additions were graphed on squared paper. Watching the progress of the maxima from group to group it was possible to guess whether a larger amplitude might be obtained for a period lying between two of the serial periods. In some such cases higher amplitudes were found, in others lower. It seems impossible by inspection of a series of graphs when slight variations in length of a period are taking place to say, without the work of rewriting the figures, which of two close periods will have the larger amplitude when the whole figures are summed. It is, however, in general possible to say for each group of years by inspection which period has the largest amplitude.

The work has not been quite uniform. Though there are 52 weeks in the year, yet, on account of the extra day in ordinary years and the second extra day in leap years, there are intercalary weeks. In dealing with the periods from 80 to 100 weeks, these intercalary weeks were omitted. For the rest of the periods, however, these were used in their proper place. In all cases the last week of the period was kept constant, the adjustment necessary because of the varying length of the periods being made at the beginning. To determine the effect of the solar year, special Tables were written out for periods of 52 and 104 weeks, the intercalary weeks being omitted in each case.

The same procedure was followed in the case of the two earlier periods, 1703–1754 and 1755–1828, with some differences. The intercalary weeks were retained in the whole of the calculations except in the Tables referring to biennial and annual periods which must be rigidly those of the solar year. No correction corresponding to the numbers of the population was possible because there are no data with which it may be effected. In addition to this variation in the number of children whose lives were at risk there has apparently been some diversity of habit in recording the deaths from time to time, a point which will be discussed later. No systematic attempt was made to determine periods over three years, as rough trials gave no evidence of the existence of any amplitude of sufficient magnitude to suggest that the application of a more laborious method would lead to any important result.

One other point remains to be stated. There are two characteristics exhibited by micro-organisms causing disease which directly concern an investigation of this nature. These are the respective powers of infectivity and of virulence. I know of no evidence that for any disease* these two qualities have any permanent relationship. An epidemic outburst may be very large and yet cause few deaths, while on the contrary it may be comparatively small and yet be associated with a very high mortality. Sometimes an epidemic is fatal at the beginning, sometimes at the end. With measles the infectivity during epidemic periods is so high that it is the general opinion of those who have conducted inquiries into this subject in schools, etc., that not more than 5 per cent. of children living in large towns fail to contract this disease. The long waves of variation in the death-rate must be thus associated with variation in the virulence of the organism.

A short note relative to the significance of the amplitude of periods found in the application of periodogram methods may perhaps be of value to those who have not been in the habit of using harmonic analysis. By this analysis the form of any periodic function is expressed as the sum of a number of terms of sines and cosines. For the simplest case the main term is of the form $\cos 2\pi x/\lambda$, where λ is the length of the period and x is the distance measured from the origin; the secondary terms are of the form $\cos 4\pi x/\lambda$, $\cos 6\pi x/\lambda$, etc. But the amplitudes found for sub-multiple periods, though large, do not necessarily signify that such periods exist in the nature of things. They may be only the result of the application of the mathematical analysis. The form of the wave even when an average of a large number of periods is taken may not be capable of even approximate representation by one term of a harmonic series. To illustrate this, four examples of series of periodic terms are given with the appropriate analysis into harmonic periods.

a . . .	2·00	2·13	2·50	3·00	3·50	3·87	4·00	3·57	3·50	3·00	2·50	2·13
b . . .	0·00	1·00	2·00	3·00	4·00	5·00	6·00	5·00	4·00	3·00	2·00	1·00
c . . .	0·00	0·00	0·00	0·00	4·00	8·00	12·00	8·00	4·00	0·00	0·00	0·00
d . . .	0·00	0·00	0·00	0·00	0·00	6·00	24·00	6·00	0·00	0·00	0·00	0·00

Choosing the origin so as to eliminate the terms containing the sines, the equations showing the harmonic analysis of these forms to six terms are respectively :—

(*a*) $y = 3-\cos x$.

(*b*) $y = 3-2·49 \cos x - 0·33 \cos 3x - 0·18 \cos 5x$.

(*c*) $y = 3-4·98 \cos x + 2·67 \cos 2x - 0·67 \cos 3x - 0·36 \cos 5x + 0·33 \cos 6x$.

(*d*) $y = 3-5·73 \cos x + 5·00 \cos 2x - 4·00 \cos 3x + 3·00 \cos 4x - 2·27 \cos 5x + 1·00 \cos 6x$.

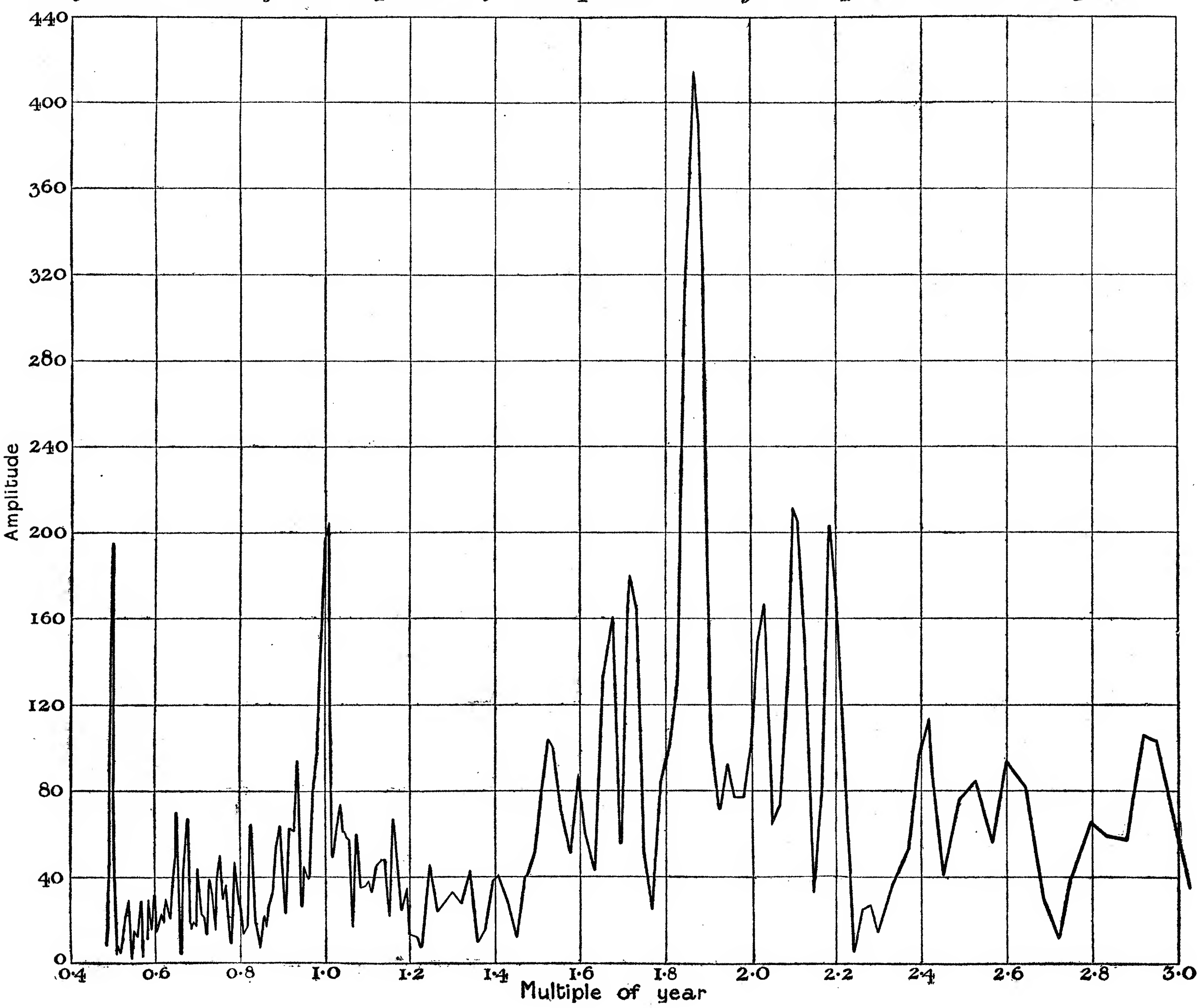
In the first two cases the amplitude of the second harmonic given by these equations (*a*), (*b*), is zero. In the third (*c*) it is rather more than half the value of the

* *Cf.* 'Journal of Hygiene,' vol. 5, p. 514.

amplitude of the chief harmonic, and in the fourth (d) nearly equal to it. When the method of the periodogram is used, similar amplitudes will be found for the half-periods as in the last two forms, but these have no necessary meaning apart from the method. Periods which are sub-multiples of definite periods found by the analysis thus require special examination before their separate existence can be held to be proved.

It seems best, in the first instance, to describe minutely the phenomena obtaining during the last 75 years, the years during which compulsory registration of deaths has been in force. Following Prof. SCHUSTER, a graph has been constructed, to show the varying importance of each period (Diagram I). In this diagram, however, the ordinates of the graph represent directly the amplitudes found for each period, and not, as Prof. SCHUSTER has done, the probability that the period has a definite

Diagram I. Showing the amplitude of each period during the epoch 1838 – 1913.



existence and is not due to chance, as I am very doubtful how far the theory of chance in its present form can be applied to the course of epidemics. These amplitudes are expressed as proportions per thousand of the mean value, above and below which the variation takes place. That is, if the curve be expressed by $y = a + b \sin x$, the amplitude is given by $1000\, b/a$. The diagram begins with a period of 25 weeks. The first point to catch the eye of the observer is the existence of a very notable amplitude in the centre of the diagram. This occurs corresponding to a period of 1·865 years, or 97 weeks. The amplitude, in the measure indicated above, is 413. This amplitude far exceeds—in fact, is double the value of—the amplitude given by any other period except those in the nearest vicinity, namely, those for 96 and 98 weeks. Had the 97-week period not varied during the 75 years in question, such high amplitudes would not have been observed, but the magnitude of the variation is no greater than might be expected in a biological phenomenon of this kind. The next point to be noted is the extent to which the numbers of deaths from measles are affected by the solar year. Amplitudes of approximately 200 are found both for the yearly and half-yearly periods. This does not necessarily mean a double period. It might equally easily mean that a single harmonic was not capable of expressing the effect of the climatic influence, though the average values through the course of the solar year of many meteorological phenomena such as temperature can be closely represented by one harmonic term.

In addition to the periods just mentioned, it is to be observed that five other periods have amplitudes varying from 160 to 210. On the diagram these group themselves in a manner suggesting interference periods. To explain this, let $y = a + b \cos mx$ represent roughly an epidemic, and let b have some form of frequency inherent in itself, *i.e.*, let $b = c + d \cos nx$. This expresses the idea already mentioned, that both infectivity and virulence vary periodically. The equation

$$y = a + c \cos mx + d \cos mx \cos nx$$

thus represents the joint effect. This at once gives rise to the form

$$y = a + c \cos mx + \tfrac{1}{2} d \cos (m+n)\, x + \tfrac{1}{2} d \cos (m-n)\, x.$$

The effect of such a double variation is thus to bring two periods which have no real existence into the range of the mathematical analysis. Such periods will be harmonically symmetrical, and will appear at certain definite relative distances from the main period, in this case 97 weeks.

Now, the periods referred to seem to group themselves in this manner, thus, the periods of 87 and $109\frac{1}{2}$ weeks, and of $89\frac{1}{2}$ and $105\frac{1}{2}$ weeks, can be explained on this hypothesis, the first being equivalent to a periodic variation of the virulence of the organism of 16 years' duration, and the second to a like variation of 22 years' duration. The coincidence is so striking that this would seem to be the necessary explanation. It might almost be said, in the language of VOLTAIRE, that, if these periods were not found, they would require to be invented.

The longer period of 114 weeks, however, is solitary. There is no period of 84·5 weeks (the number 84·5 corresponding to that of 114 weeks) found to possess an amplitude equivalent in magnitude to that of the latter period as is required by the interference hypothesis just discussed. It must, however, be examined to see whether this period might result from an interference between the great 97 weeks period and that derived from the climatic variations of the solar year. This calculation gives a negative result. Such a period must be placed about 112·4 weeks, with a corresponding period of about 34 weeks. The difference between 112·4 and 114 is too great to allow of the two periods being considered identical. This, however, does not negative the existence of a climatic influence on the epidemic of measles. The amplitude of the period of 34 weeks is only 43, a very small number in comparison with those in question, but comparable to that found to exist for the period of 112·4 weeks. The period of 114 weeks, however, cannot be explained by interference, and must be considered as possessing an independent existence.

The form of the curves obtained by the periodogram averages is of special interest, as these must represent smoothed epidemic outbursts. The amplitudes of the first three harmonics of the chief periods found are given in the following Table (Table I).

TABLE I.—Amplitudes.

Period in weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.
87	161	18·0	12·8
89½	181	18·9	8·6
97	417	94·2	26·0
105	150	205·1	48·3
106	168	49·8	67·8
109½	212	59·8	27·0
114	211	36·4	39·0

The sums of the columns of figures are for the first two periods surprisingly close to sine curves. For the main period the amplitude of the second harmonic is about one-fourth that of the chief harmonic, and much the same holds for the last two periods. One exception exists, that for 105 weeks, in which a very high value for the amplitude of the second harmonic is found. As this sub-multiple period is equal to 52·5 weeks it obviously is in such close proximity to the annual period as to make differentiation difficult. The true period probably lies between 105 and 106 weeks. When the latter is considered, it is found that the amplitude of the second harmonic bears the same relationship to the amplitude of the first as is generally found. This exception thus seems an accidental phenomenon.

It will be seen later that there is no evidence for the existence of long waves of

16 and 22 years required to account for the pairs of periods already found from the interference theory. But the hypothesis must be rejected for other reasons. As already stated, the additions of the weekly columns were not only made for the complete 75 years, but for four approximately equal groups, each including sets of from 18 to 19 years. With regard to the six special periods 87, $89\frac{1}{2}$, 97, 105 and 106, $109\frac{1}{2}$, and 114 weeks, the main amplitudes have been calculated for these groups of years severally. If the harmonically symmetric pairs were the result of the same special influence, the amplitudes should show the same kind of variation from group to group of years for each symmetric pair.

If the Table (Table II) be examined, it will be seen that the variations found to exist do not show this correspondence.

TABLE II.—Showing the Amplitudes in the Groups of Years and for the whole Epoch of the Chief Periods Found.

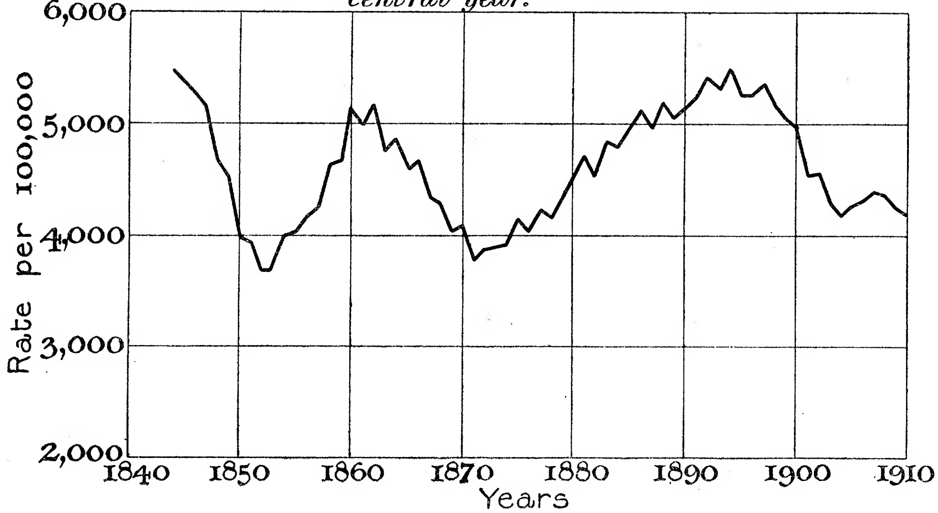
Groups of Years.	Number of weeks.							
	87	89	$89\frac{1}{2}$	97	105	106	$109\frac{1}{2}$	114
I.—1838–1854	110	231	234	272	40	284	331	270
II.—1854–1874	152	35	45	552	306	219	70	208
III.—1874–1893	303	300	302	416	25	82	229	110
IV.—1893–1912	310	180	132	464	122	167	325	397
Whole epoch	161	167	181	417	150	168	212	205

When the period of 87 weeks is compared with that of $109\frac{1}{2}$ weeks, the amplitudes, except in the last case, show no evidence of mutual dependence. The same is the case with the periods of $89\frac{1}{2}$ and $105\frac{1}{2}$ weeks. Such differences make it somewhat difficult to assign the periods to the same causes. This difficulty is increased in the present case when it is noted that the chief correspondence is found in the last two groups, which is in contradiction to the evidence given by the diagram which describes the variation of virulence. During this portion of the epoch, as will presently be seen, there is a complete absence of evidence of any periods in the neighbourhood of 17 and 22 years. In other words, the influences which make the amplitudes of one of the periods notable in one group of years are not even approximately the same as those which determine the amplitudes of the assumed corresponding period during the same group of years. The harmonic symmetry must, therefore, either be considered the result of coincidence, or remain for the moment unexplained.

The amplitudes found in the longer periods, namely, those exceeding three years, now fall to be explained. The manner in which fatality has varied may perhaps be best seen by the use of a Bloxamed curve. Such a smoothing of statistics is at best a

makeshift. When, however, a disease has a periodicity of nearly two years it is evident that the yearly death-rates must oscillate largely, and that this oscillation, if plotted as a graph, must obscure to a considerable extent the variations of the fatality of disease. The statistics have, therefore, been Bloxamed in nine yearly averages. The result is represented in Diagram II. Reference to this diagram

Diagram II. Showing the death rates from measles per 100,000 of children under five years, Bloxam in nine yearly periods of which the year shown is the central year.

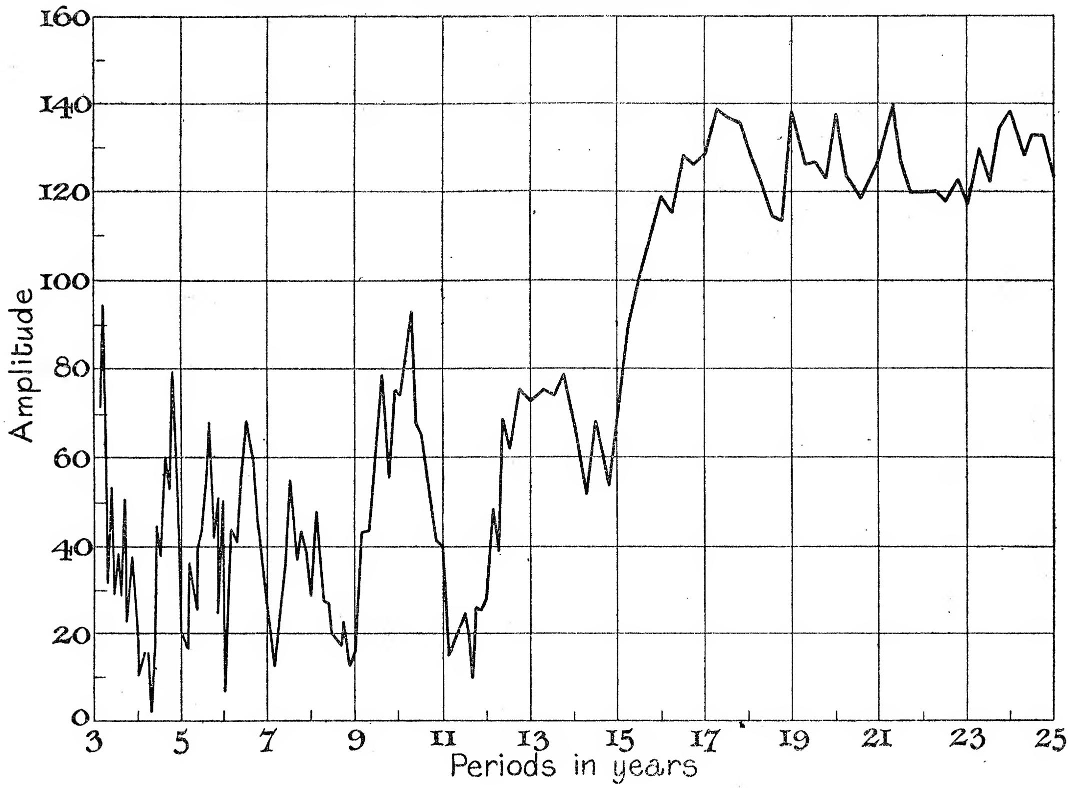


shows that the mortality of measles was low at two separate epochs, namely, about 1852 and 1871. There is, however, no sign of a third minimum about 1890. Such a minimum does not appear till 1904. Long periods with large amplitudes will thus be non-existent, but moderate amplitudes may be expected to exist for a large range of periods. The application of the systematic method shows this to be true (Diagram III). As stated before, the amplitudes have been investigated for every quarter-year period, beginning with three years and extending up to 25 years. The amplitudes at first are very small. When a period of $10\frac{1}{4}$ years is reached the amplitude has risen to 93. Thereafter, it again falls nearly to zero at 11·6 years. From this point it slowly rises to a limit of 70, at which it remains till the period is 15 years. A gradual increase leads to a level of about 125, on the surface of which several small waves appear. For the final period of 25 years the amplitude is 113. The only definite maximum is that at 10·25 years. This period has been examined in conjunction with the 97 weeks period, without, however, the discovery of amplitudes of any size at the points where interference periods might be expected, namely, at 1·574 and 2·272 years.

These findings, however, are in accordance with the results already obtained.

The conclusion seems to be that there is no evidence for any constant long waves in the virulence of the measles organism so far as the limited amount of information at our disposal permits an opinion to be formed.

Digram III. Showing the amplitudes of the periods between three years and twenty-five years for the epoch 1830–1914.



A reference must now be made to some other points in connection with the nature of the periodicity. With regard to the 97-week period it is found that this has varied slightly in length. In the first group of years the highest amplitude occurs at a period of 95 weeks, and amounts to 301 as against 272 for the 97 weeks period. Similarly for the third group the amplitude at 96 weeks is 455 as against 416 for the 97 weeks period. In the second group of years the period is more nearly 97·5 weeks. These variations cannot, however, be considered more than might be expected.

Some more important features remain to be noted. In the first group of years, and in the last group, the amplitudes of the 97-week and of the 114-week periods are large and nearly equal. No evidence of any kind has been found to suggest that there is anything but complete independence between these periods. If this independence is a reality, it follows that two epidemics of different periodicities have been running in London together for the last 75 years, though for a considerable period one of them has been more or less in abeyance. If this intermixture of epidemics is admitted it increases the likelihood that the cause of an epidemic is associated in some way with the variation in the power of infectivity on the part of an organism. When this infectivity rises, other things being equal, an epidemic is the result. The facts thus suggest the existence of two different strains of the measles organism having somewhat different peculiarities in their life-histories.

It is difficult to see how regular double periods can be explained on the hypothesis of constant infectivity. In the interest of this assumption it might be predicated that the different periods are the result of difference of class conditions which distinguish the different districts of London. But in this connection it must be remembered that it is only in poorer districts that measles, as a rule, is associated with a high mortality, and that consequently the statistics must largely refer to such districts. During the last 20 years the 97-week and the 114-week periods have been of nearly equal amplitudes, so that the difference between the periods can hardly be explained on the latter hypothesis. Further, if the periods associated with the other high amplitudes are really independent, it seems impossible to explain the constancy of periods found during the last 75 years, years in which great changes of all kinds have taken place, by simple variation in the number of susceptible persons.

The only other phenomenon remaining to be discussed for this 75 years is the mode of action of the seasons of the year on the course of the measles epidemics. It is better to delay this until the whole data have been examined.

The Weekly Bills of Mortality contain all the information for the years prior to 1838 which is extant. Unfortunately, these data cannot be said to possess the same authority as the statistics available since the introduction of registration. Various factors tend to limit their value. The record of the epidemics of measles does not seem to have been kept on any regular system. Prior to the year 1703, measles as a cause of death was included in a joint group with small-pox. For a considerable number of years after this date, though measles was distinguished separately as a cause of death in the Bills of Mortality, it must have happened that either large numbers of deaths due to measles were still returned under the heading of small-pox, or that epidemics of measles were only occasionally of a severe type. This conclusion follows from a consideration of the intervals which exist between succeeding epidemics. In addition, it would seem that at times a number of deaths, which should have been distributed over a series of weeks, are collected into one entry. Thus, for example, an entry of 100 deaths, though this is an extreme case, is given as occurring in one week, though the number of deaths in the weeks immediately preceding and following are barely one-fourth of this. That the figures are not a misprint is shown by the fact that the total for the year corresponds to this entry. It is, of course, possible that a return from some large children's institution, such as the Foundling Hospital, might furnish the majority of these deaths, or that, on the other hand, there may have been an error in the original manuscript. Further, some of the epidemics recorded are exceedingly large. When this happens, the results derived from the periodogram analyses, when the period is long, must be carefully questioned. In such a case, with each increase in the period, the number of the rows in the columns is reduced. Consequently, an epidemic of exceptional size tends to unduly increase the values of the columns in which the figures belonging to the epidemics appear. Large amplitudes may thus be observed

corresponding to periods the existence of which is open to legitimate suspicion. The statistics, however, are unique, and a complete investigation is therefore required. This investigation has been carried out, as already stated, in two stages: (1) for the epoch 1703–1754, and (2) for the epoch 1755–1828. The break at 1828 is due to the fact that the figures for one year, including a small part of 1829, and nearly the whole of 1830, are unfortunately wanting. The epoch 1831–1837 is obviously not in itself long enough to repay study.

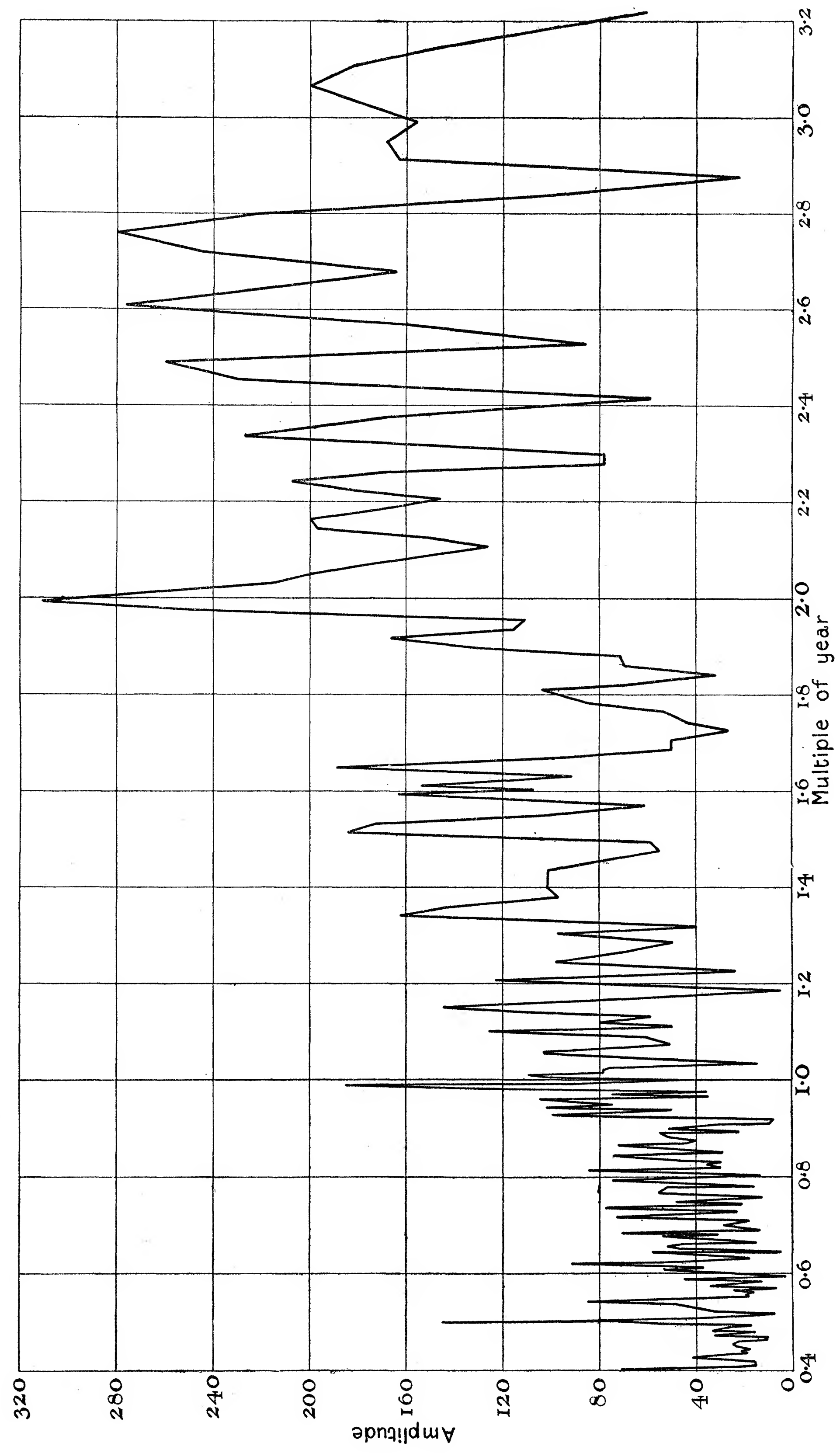
The results for the second epoch are considered first; when the graph for this epoch is examined and compared with that for the epoch 1838–1913 (Diagram IV), surprising differences are found to exist. For the years 1755–1828 the main period is that of two years exactly, or perhaps very slightly less. No evidence of a period in the neighbourhood of 97 weeks is to be found. The annual and semi-annual periods are also not nearly so prominent. The period of 114 weeks, however, has not necessarily disappeared. The amplitudes for the periods of 112 and 113 weeks are respectively 197 and 199. This difference between 112·5 and 114 weeks is not beyond the variation already found in the examination of the figures for the sub-epochs between 1838 and 1914. During that epoch the 97 weeks period varied from 95 to 97·5 weeks. In like manner, the 87 weeks period may be represented by a period of 86 weeks. Large amplitudes are found for several long periods—117, 122, 130, 136, and 144 weeks—periods for which no evidence exists in data since the year 1838. The whole characteristics of the graph of the amplitudes are thus quite different from those observed for the epoch 1838–1913.

With regard to periods which seemed fairly definite in the epoch 1838–1913, it was observed that, generally, the amplitude of the second harmonic did not exceed in amount one-third of the amplitude of the first harmonic. Selecting and tabulating the chief periods in this epoch and their amplitudes (Table III), it is found that the same rule holds. In addition, with the period of 86 weeks in this epoch, a similar prominence of the amplitude of the first harmonic, which characterised the period of 87 weeks found in the later epoch, is also observed. It is thus possible that the

TABLE III.—Amplitudes.

Period in weeks.	1st harmonic.	2nd harmonic.
86	188·7	37·6
100	166·5	105·4
104	291·9	101·1
112	196·9	66·9
113	198·9	57·8
117	207·6	79·5
122	236·5	76·4
130	260·1	97·9
136	276·3	97·9
144	279·4	98·6

Diagram IV. Showing the amplitude of each period during epoch 1755 - 1828.



87-week period is represented during the epoch 1755–1828 by a period of 86 weeks. The 114-week period is also probably represented by the period of 112·4 weeks, but of the presence of none of the other periods, including the great period of 97 weeks, is any sign to be observed. The differences found are, however, not so singular as might appear at first sight, though the significance of the findings can only be discussed in relation to the phenomenon observed in other districts, for which sufficient data have been examined.

The figures for the epoch between the years 1703 and 1755 are still more imperfect than those for the epoch just investigated. It must also be noted as a great drawback that a single epidemic of measles occurring in the year 1742 caused an enormous number of deaths, amounting in all to nearly one-eighth of the total recorded for the whole epoch of 53 years. The figures for this epidemic might have been excluded, but it has been thought better to carry through the work on the same lines as for the later epochs, and then to estimate how far the results obtained by this method may have been vitiated by the presence of the exceptional epidemic. No search for periods beyond the limit of 160 weeks has been attempted.

The graph of the amplitudes constructed from the figures calculated sufficiently illustrates the difficulties of interpreting the statistics (Diagram V). It does not seem probable that such a large number of periodicities as are shown there can be accepted as existing without more extensive evidence. Even in the case where exceptionally large amplitudes are present, the periods are not defined nearly so sharply as between the years 1838 and 1913. It appears, therefore, more likely that some accidental concurrence of large epidemics of different origin in the tabulation has tended to produce the phenomena observed.

In this epoch for the first time a number of periods of less than 80 weeks with considerable amplitudes, that is amplitudes exceeding 200, are found to exist. These periods are, however, without exception second harmonics of longer periods, and consequently their independent existence must be considered doubtful. Some of these in themselves are, however, not inherently improbable, as will be seen in a later part of the discussion.

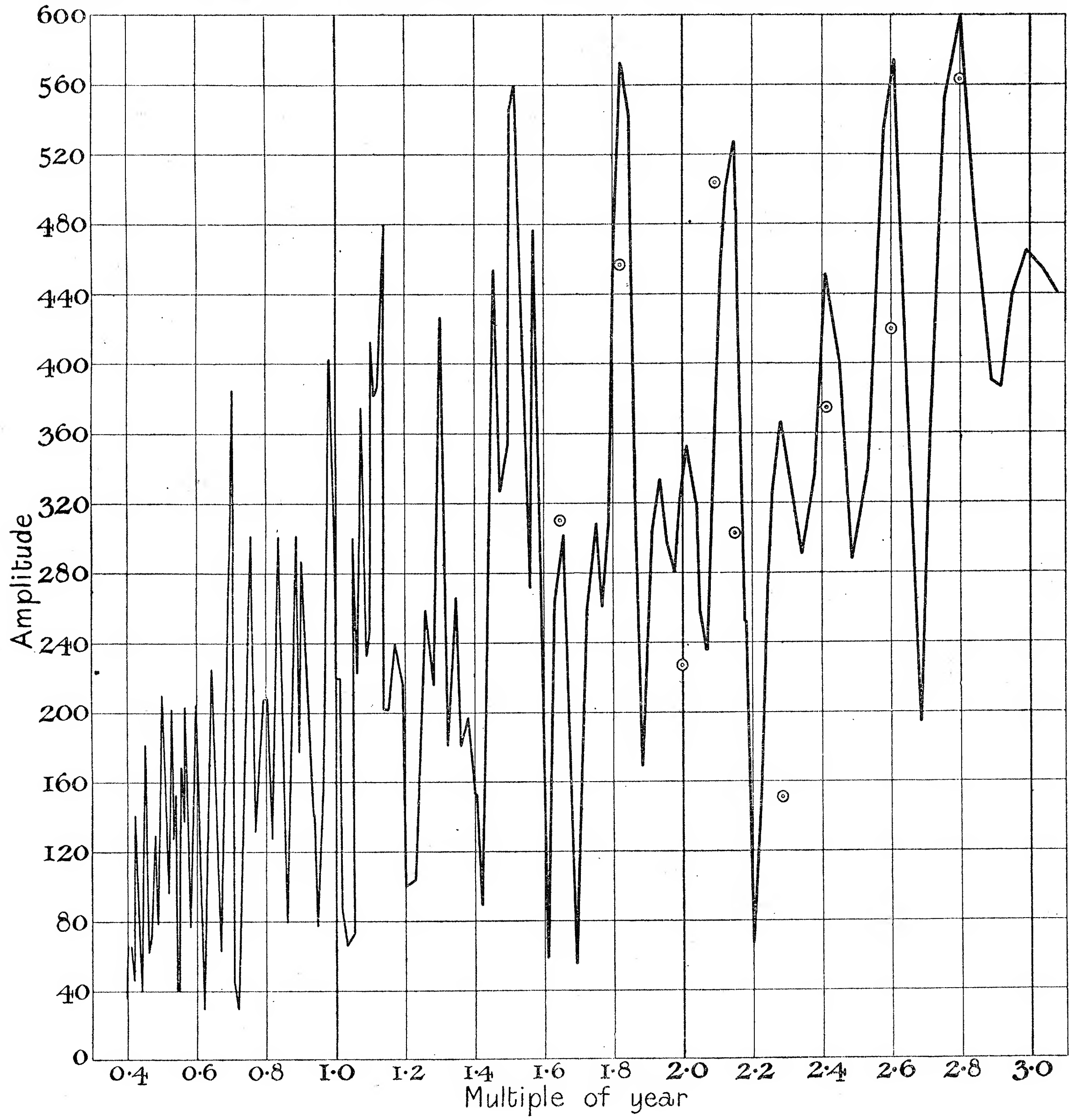
A large amplitude is found for the period of 95 weeks, a period which suggests the great period of 97 weeks obtaining in recent years. The variation from this latter is not greater than that already found to exist for the sub-epochs of the epoch 1838–1913. It must, however, be remembered that no evidence of this period is found between the years of 1755 and 1828.

Large amplitudes are also found for a number of periods exceeding two years in duration, and some of these may have their counterpart in the periods found for the middle epoch.

As already remarked, one very large epidemic played the part of a disturbing factor. For the periods with the larger amplitudes a re-calculation of the amplitude has been made after the figures relating to this epidemic have been subtracted. The

results are shown in Diagram V. In this diagram the values of the recalculated amplitudes are indicated by a dot surrounded with a circle. In some cases the amplitudes are greatly diminished in amount but in others no serious difference between the old and new values has been discovered. The amplitude of the 95 weeks

Diagram V. Showing the amplitude of each period during the first epoch 1703-1754



period remains almost unaltered; the period of 111 weeks is rendered much more prominent than that of 109 weeks; the period of 119 weeks disappears; those of 126 weeks and of 136 weeks have their amplitudes somewhat diminished, while the period of 146 weeks, which possesses the largest amplitude of any found in the

investigation, remains practically unaltered. It would thus seem that a number of definite periods exist of which there is neither counterpart in the epoch between 1755 and 1828 nor in the epoch since the introduction of registration.

A further difference also demands special attention. The considerable amplitudes associated with the annual and semi-annual periods which were a feature of the statistics for the last 75 years and which were much diminished during the epoch 1755–1828 again make their appearance. Considering the general character of the graph, however, the significance of their presence can hardly be considered as of the same interest as for the epoch described in the more accurate data of registration. From the data for the epoch 1703–1754 considered purely by themselves it is difficult to draw any definite conclusions, but, as will be seen later, they afford some confirmatory evidence which is not without importance to epidemiological theory.

It now remains to consider how far the results of this investigation conform to or are explained by the facts observed in other places. The most important independent evidence refers to Scotland. Since the year 1855 the Registrar-General for that country has published the monthly number of deaths from measles for each of the eight chief towns. These have been investigated for Glasgow, Aberdeen, Dundee, Perth, and Paisley. Additional information for the first of these is furnished in Dr. WATT'S celebrated 'Treatise on the Chincough,' for the years between 1780 and 1820. The monthly numbers of deaths from measles for the last 30 years have also been extracted from the statistics of Paris. No attempt has been made to examine all the existing periods. In all these cases the amplitude of one particular period so greatly exceeds the amplitude of any other that this special period must be considered predominant.

When the statistics of these different localities are examined, it is found that in nearly every case a different epidemic periodicity exists, a fact that must be considered of fundamental importance. The shortest period found is that existing in Paris during the last 30 years, which is accurately, or almost accurately, that of the solar year. No other period has been discovered to exist in these statistics. As the figures for this city have not been published in either of my previous papers they are added in Table IV. Nearest to this period in duration is that found for the city of Glasgow between the years 1773 and 1806, a period of very nearly 67·3 weeks. Closely associated in length is the epidemic periodicity found to obtain in Perth between the years of 1871 and 1911, which amounts to almost exactly 69·3 weeks. Both of these latter periods are shorter than any observed in London, except during the first epoch. Following these periods in duration, though the significance of the figures is not quite clear, is the 78-week period found present in Dundee between the years 1856 and 1873. The amplitude of this period is very large, amounting to about 600, but, for a reason to be given later, the existence of this period is questionable. The exact period of two years is frequent. It existed in Paisley between the years of 1856 and 1869; in Dundee, from 1874, and in Aberdeen, from 1892 to the

TABLE IV.—Deaths from Measles, Paris, 1880–1910.*

Year.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.	Total.
1880	25	37	83	68	122	156	150	113	61	40	59	72	986
1881	101	92	85	77	93	93	131	53	46	48	49	57	925
1882	67	98	126	140	153	122	85	57	31	25	48	66	1018
1883	71	71	105	140	135	124	116	103	48	25	47	82	1067
1884	73	103	179	197	181	141	114	93	89	99	123	141	1533
1885	164	156	185	234	202	169	106	65	42	65	97	79	1564
1886	95	48	113	131	129	110	137	90	60	74	93	175	1255
1887	197	186	256	276	224	179	103	67	53	33	42	58	1674
1888	57	70	70	51	70	78	96	95	52	46	95	178	958
1889	207	152	158	141	95	96	89	63	43	40	62	74	1220
1890	54	89	155	185	230	272	230	92	58	41	57	69	1532
1891	66	65	116	184	176	171	114	54	27	7	13	27	1020
1892	47	56	114	141	161	134	91	72	29	18	17	39	919
1893	47	24	43	81	97	157	90	63	20	19	11	25	677
1894	71	78	101	169	227	159	97	52	13	11	8	7	993
1895	4	20	24	32	64	93	138	87	49	22	70	76	679
1896	121	113	86	69	71	52	55	34	22	6	11	18	658
1897	57	79	96	195	151	94	68	27	2	6	16	30	821
1898	38	72	105	151	167	140	113	61	10	3	6	10	876
1899	33	79	98	137	142	126	103	111	30	15	14	21	909
1900	53	86	121	128	120	113	132	48	15	10	12	16	854
1901	30	31	48	70	75	90	62	52	14	16	27	30	545
1902	51	55	85	102	112	128	68	21	21	4	14	14	675
1903	26	36	61	64	51	50	49	31	24	17	16	21	446
1904	63	67	68	77	93	73	70	32	9	6	10	18	586
1905	19	19	47	38	51	49	54	24	20	12	31	60	424
1906	97	83	107	96	66	40	38	6	4	7	9	19	572
1907	37	35	45	54	58	68	51	43	8	11	5	18	433
1908	45	56	85	99	91	62	26	10	8	11	3	11	507
1909	7	17	43	47	62	67	80	48	25	34	27	41	498
1910	60	93	93	122	137	105	57	29	17	10	7	7	737
Totals. . .	2083	2266	3101	3696	3806	3511	2913	1796	950	781	1099	1559	27,561

* 'Annuaire Statistique de la Ville de Paris.'

present day. This period corresponds to the main period observed in London during the middle epoch. In Glasgow, on the other hand, a two-yearly period does not seem to have been present in recent years, the maximum amplitude being associated with a period of 106·5 weeks, though there is also apparently during the last 25 years a subsidiary period of approximately 102 weeks. The Glasgow statistics deserve a more complete investigation than they have hitherto received, but even with the present information they afford quite sufficient evidence to show that the exact two-yearly period shown to exist in a number of places has no essential connection with the solar year. For longer periods it is to be noted that in Aberdeen between 1856 and 1885 the epidemics of measles occurred at intervals of 143 weeks. This is the longest undoubted period discovered, nor on investigation has any subsidiary period of importance been found to exist. The amplitude of this period exceeds 800. The existence of this periodicity in Aberdeen for 30 years demonstrates the probability that some of the long periods found by the present method to exist in London prior to the introduction of registration may represent concrete facts. In this connection the period of 146 weeks found to be the main period in the first epoch must be mentioned. With regard to Dundee, I considered in my previous papers that for the first 20 years of registration the intervals between the epidemics should be considered as three years. This opinion I still hold. As the point is important, the figures are reprinted (Table V).

TABLE V.—Deaths from Measles in Quarter Years, Dundee. (Eighteen years, 1856–1873, in three-year Periods.)

	17	106	47	2	2	—	1	—	—	5	10	78
	103	12	2	1	*2*	*13*	*35*	*29*	*9*	*2*	4	18
	118	97	29	12	*8*	*4*	*46*	*43*	*16*	*7*	2	—
	1	71	103	15	6	1	—	2	1	2	1	1
	75	104	6	—	1	—	1	—	—	1	5	20
	11	8	—	*2*	*3*	*42*	*76*	*12*	*8*	*9*	*7*	1
Totals	325	398	187	30	9	1	2	2	1	8	22	118
	—	—	—	*2*	*13*	*59*	*157*	*84*	*33*	*18*	*7*	—

It will be seen that this interpretation of the statistics implies that two sets of epidemics, each having a three-yearly period, were running contemporaneously. With a length of period so great as three years in a disease which chiefly attacks young children, it is obvious that, at the date at which each new outburst of disease was due, there would be a very large susceptible population: subsidiary epidemics would, therefore, be easily produced. This explanation, in view of the sequence of the figures, seems to me the most probable, though, as before remarked, with a period of 18 months the amplitude is greater than 600. The facts just described throw considerable light upon the interpretation which must be put upon the

statistics of London. It is quite obvious that no period found in London need be rejected because of its duration. It is also proved by several undoubted instances that no periodicity has any elements of absolute permanency. The changes observed in London are therefore no more than might be expected from the evidence obtained from other districts. Even the results of the investigation of the first epoch, doubtful though they are, may contain a very large percentage of truth.

One point, however, of the greatest importance remains to be discussed, and that is the influence of the climatic changes of the year. In the description of the epidemiology of measles in London it is constantly remarked that there are two well marked seasons in the year at which an excess of deaths from measles may be expected. From this statement it has been assumed that a like phenomenon exists generally, but this is certainly not the case for Paris, Aberdeen, Dundee, or Glasgow. Whether it is a unique phenomenon or not I am unable to say at this stage of the enquiry. Further, if the climatic variations of the solar year have an effect in determining the number of deaths from measles, the effect should be seen by the appearance of interference phenomena to be observed in the graph of amplitudes. Now, there is no evidence, when the amplitude found to correspond with the solar year is considered, that the solar year produces interference phenomena with the main period of 97 weeks at all comparable to its importance. The amplitude of 60 observed at the symmetric corresponding periods of 34 weeks and 112·4 weeks are far in defect of what might be expected from the interference of periods with amplitudes respectively 200 and 417. Further, in the middle epoch the amplitude for a period of exactly 52 weeks is very small compared with that found in the preceding and succeeding epochs. This suggests that the source of the amplitudes associated with the yearly period is to be sought rather in the organism of measles than in the climatic variation of the seasons.

Pursuing the same process of reasoning, it follows that the fact that the epidemic period in Paris is exactly, or almost exactly, that of the solar year may be no more than the expression of a coincidence between the length of a stage in the life-history of the measles organism and the length of the solar year. In this connection there is some evidence that in London epidemics with periods slightly greater or slightly less than the solar year exist. Thus, in the epoch 1838–1913, the largest amplitude is not for the exact solar year, but for a period slightly greater, as may be seen in Diagram I; while for the middle epoch, though the amplitude for the exact solar year is negligible, yet for a period of four days less an amplitude of 180 is found. A large amplitude in the same position is found also for the first epoch. Thus, there is good evidence for the continued existence of a period of very slightly less than one year during more than a century. Biennial epidemics may have existed, to some extent, in the first epoch, but the chief evidence for their existence in London is found in the second epoch, to which they contribute the main epidemic periodicity. They are, however, almost completely absent during the last 75 years. A large

amplitude is found for a three years period in the middle epoch, and a considerable amplitude in the earliest epoch, but, again, no evidence for such a period exists during the last 75 years. Considering the whole of the facts, it would seem that the influence of the changes in the solar year in determining epidemics of measles is not nearly so direct as has been supposed.

The almost exact coincidence of the periods in many places with one year or two years suggests, however, that the variation of the seasons may have played a part in the evolution of the life-history of the measles organism, and that, at the present moment, the apparent influence of the solar year is the result not of present but of past action.

Conclusions.

The conclusions which may be based on the results of this investigation cannot in the present state of knowledge be completely formulated. Much more spade-work is required before the relative importance of the different factors at work can be accurately gauged. It seems to be proved, however, when the statistics of towns or cities are severally examined, that epidemics of measles tend to recur with a regular periodicity, which persists often for a long series of years. A great variety in the length of these periods has been found to exist, the shortest period found being that of one year in Paris, and the longest that of three years in Dundee. That the length of the period has nothing to do with the size of the town nor with the rate of increase in the population, may also be taken as demonstrated. Thus, in Glasgow, at the end of the eighteenth century, the epidemic period was 67·3 weeks, while from 1855 to the present day the chief period found is quite definitely 106·5 weeks. On the other hand, in Aberdeen and Dundee the epidemic periods were 143 and 156 weeks respectively in the early years following the introduction of registration, while in both of these cities the period has been 104 weeks in more recent years. That the change in both these cases from a longer to a shorter periodicity was sudden, and not gradual, is a matter of direct observation. The disappearance of the 67·3-week period in Glasgow was also sudden, but, owing to the change occurring towards the close of the epoch for which the early statistics exist, it is not possible to determine the period which immediately succeeded. In none of the Scottish towns is there any evidence that two epidemics of different periodicities have run concurrently except in the case of Glasgow during the last 25 years, but as the two periods differ as is before shown by only 4·5 weeks, and as one of the epidemics is much larger than the other absolute proof is impossible.

The statistics of London illustrate and confirm these statements. The rapid changes in the nature of chief periodicities can be easily seen. Between 1755 and 1828 no evidence of any period comparable to 97 weeks can be found. By the year 1838 this period is well established and has existed ever since. But the statistics of London

demonstrate also that two or more epidemics with different periodicities may run concurrently. Any but a biological explanation of these phenomena seems to me untenable, and the facts further seem to prove that the periodicity is infinitely more probably due to such changes as may easily constitute the life-cycle of an infecting organism rather than to periodic changes in the susceptibility of the host. To explain the phenomenon by the number of susceptible persons present in the population at any one time seems hardly possible.

The influence of the climatic variation of the solar year has had more darkness than light thrown upon it by the present investigation. That there is a seasonal influence is undoubted but the relationship is not simple as has been commonly supposed, but much more complex. A considerable part of the influence seems to be due to the fact that it is not unusual for the complete cycle of the life-history of the measles organism to correspond almost exactly with a multiple of the solar year, in itself a suggestive coincidence, but even if this has been fully allowed for something remains which requires further analysis. I hope some day to discuss these problems, but when it is remembered that, even with a disease such as summer diarrhœa, in which the dependence of the size of the epidemic on the degree of summer temperature is obvious, no one has succeeded in demonstrating any simple connection, it is possible that the explanation of the relationship of epidemics of measles to the solar year may yet be far in the future.

APPENDIX.

TABLE I.—Showing Amplitudes of the Harmonics of the Different Periods, London, 1838–1912.

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
80	101·4	36·4	4·7	
81	70·8	9·3	11·5	
82	57·7	48·7	22·3	
83	88·9	31·3	22·6	
84	59·0	13·4	28·9	
85	43·6	17·3	3·0	
86	136·2	64·9	14·0	
87	161·5	18·0	12·8	
88	56·8	7·5	29·5	
89	167·4	22·7	3·9	
$89\frac{1}{2}$	180·3	19·0	9·3	
90	164·5	26·0	11·5	
91	51·5	33·1	30·8	
92	25·1	55·2	14·8	
93	84·0	62·7	32·3	
94	135·4	29·8	16·6	
95	133·2	63·7	18·7	
96	314·1	62·9	23·4	
$96\frac{1}{2}$	387·1	63·2	21·1	
$97\frac{1}{3}$	417·0	94·2	26·0	
97	393·9	82·1	30·2	
98	319·8	16·1	38·6	

TABLE I—*continued.*

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
99	103·4	45·6	20·1	
100	71·2	39·3	36·5	21·9
101	93·4	79·0	49·1	8·4
102	77·4	98·7	43·1	16·16
103	77·5	150·7	4·1	47·9
104	122·5	174·4	22·5	221·1
105	154·8	208·9	36·7	49·8
106	167·5	49·8	67·8	18·7
107	82·5	65·8	47·6	8·3
108	72·2	74·8	19·7	4·7
109	188·5	66·1	16·7	9·4
109½	212·3	59·8	27·0	15·3
110	206·4	62·2	44·8	8·3
111	132·9	15·9	29·7	33·3
112	32·5	60·2	21·3	
113	161·4	35·8	13·6	
114	205·1	32·7	11·9	
115	162·5	38·1	23·8	
116	76·2	33·2	15·5	
117	5·2	44·3	41·6	
118	24·9	48·0	50·3	
119	27·1	48·5	30·3	
120	14·7	22·7	25·6	
122	38·5	67·3	12·7	
124	52·7	25·0	33·6	
125	97·6	35·9	22·2	
126	113·4	13·2	22·8	
127	89·3	12·6	17·8	
128	41·4	7·0	11·8	
130	76·9	46·6	42·4	
132	85·2	24·6	22·1	
134	55·6	29·5	23·5	
136	94·0	33·6	17·8	
138	83·7	28·5	62·9	
140	30·8	43·4	65·4	
142	10·4	9·6	17·3	65·3
144	40·7	16·7	76·0	
146	66·3	39·4	101·2	
148	59·9	41·3	70·5	
150	58·4	28·4	7·2	
152	106·0	14·1	87·6	
154	103·1	41·7	77·3	
156	68·5	52·6	151·1	
158	35·3	82·9	166·8	
160	35·2	104·1	59·0	

TABLE II.—Showing the Amplitudes of the First Four Harmonics for Quarter Years, 1840–1914.

Quarter years.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
50	71·2	40·9	21·7	72·2
51	75·3	54·2	2·4	95·0
52	72·6	68·3	23·6	39·3
53	75·0	59·0	38·2	30·5
54	73·5	45·6	40·3	53·7
55	79·1	41·7	46·9	28·7
56	66·7	24·3	47·6	36·5
57	50·3	12·3	54·4	27·8
58	67·2	23·7	35·5	48·0
59	53·3	34·4	28·5	51·0
60	70·4	53·6	6·7	22·4
61	89·3	36·1	34·5	34·4
62	101·9	43·4	27·1	37·1
63	109·9	38·9	39·4	25·3
64	118·0	28·2	55·3	21·1
65	114·5	47·8	51·6	11·4
66	127·0	26·9	52·7	13·5
67	125·9	26·1	57·9	27·4
68	128·7	19·2	53·6	14·7
69	138·6	17·0	45·3	2·2
70	136·7	22·0	39·3	18·2
71	136·1	12·9	32·2	44·3
72	129·3	16·1	11·3	37·4
73	123·8	42·9	8·7	36·9
74	114·3	43·4	25·7	59·3
75	113·4	47·3	46·1	51·6
76	137·7	68·3	69·8	79·3
77	126·0	78·5	45·2	47·5
78	127·1	54·3	75·0	37·7
79	123·0	75·0	62·3	35·5
80	137·9	73·7	43·3	26·5
81	122·5	85·3	64·8	18·7
82	118·4	92·7	42·9	15·7
83	122·9	68·2	37·9	37·0
84	128·4	64·9	28·9	29·8
85	139·5	58·2	9·7	24·6
86	126·7	45·5	7·7	41·0
87	118·9	40·9	7·6	44·4
88	119·6	40·4	26·5	48·6
89	119·9	13·5	19·0	54·0
90	116·8	18·2	23·4	68·4
91	122·4	24·8	29·1	54·3
92	115·9	17·8	53·1	42·3
93	129·7	9·3	26·9	49·1
94	121·6	25·6	32·1	24·8
95	135·3	25·9	45·3	52·3
96	138·7	28·6	28·3	5·0
97	127·9	47·4	29·0	20·1
98	133·3	39·2	28·7	21·7
99	122·9	68·6	13·3	5·0
100	113·4	61·4	5·7	41·6

TABLE III.—Showing the Amplitudes of the First Four Harmonics in each Period, 1755–1828.

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
84	154·2	56·2	58·6	71·0
85	91·2	94·7	65·4	24·2
86	188·7	37·6	42·8	15·1
87	99·4	48·4	24·9	16·4
88	50·0	74·8	25·4	22·0
89	50·6	17·9	17·6	41·8
90	27·4	71·6	17·0	35·5
91	53·7	49·0	9·8	18·7
92	57·4	54·9	45·5	21·9
93	85·2	56·6	8·7	17·0
94	112·2	51·7	30·7	23·1
95	70·4	7·9	53·6	24·8
96	32·8	4·2	43·6	21·3
97	70·0	100·4	91·9	11·0
98	70·6	49·5	21·3	10·3
99	131·2	73·7	19·7	32·8
100	166·5	105·4	52·3	15·1
101	115·7	35·7	5·2	33·3
102	110·7	37·4	55·7	26·8
103	248·6	185·3	51·8	17·0
104	291·9	101·1	14·9	54·5
105	264·4	123·3	33·3	72·7
106	215·6	77·1	55·6	40·3
107	197·3	74·9	71·2	22·8
108	176·4	13·5	36·2	7·7
109	149·7	62·3	90·8	39·4
110	126·0	113·3	61·1	55·3
111	151·2	86·0	6·2	40·5
112	196·9	66·9	72·8	47·9
113	198·9	57·8	37·2	82·6
114	171·8	64·2	24·0	57·6
115	145·8	125·7	78·2	18·0
116	180·2	50·0	20·3	18·7
117	207·6	79·5	52·1	16·9
118	166·8	58·0	36·0	22·1
119	77·6	119·0	34·6	5·8
120	78·1	145·3	57·6	32·1
122	236·5	76·4	15·0	12·4
124	169·3	5·3	77·3	3·1
126	59·4	123·8	13·9	36·6
128	229·2	22·7	36·9	37·9
130	260·1	97·9	29·2	51·3
132	89·2	70·8	66·7	18·6
134	160·7	49·8	53·3	58·0
136	276·3	97·9	54·1	47·1
138	221·6	39·9	52·9	45·7
140	164·5	162·3	21·1	34·0
142	248·5	141·8	30·3	29·9
144	279·4	98·6	8·7	12·6
146	222·7	103·8	88·7	24·2
148	100·3	101·7	102·2	17·8
150	23·7	101·3	84·7	51·6
152	163·0	78·3	75·0	22·9
154	168·3	55·4	136·8	50·7
156	153·2	59·6	91·8	48·2

TABLE III—*continued.*

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
158	180·2	184·2	129·1	11·6
160	197·7	173·4	78·6	55·9
162	172·0	104·6	7·3	51·0
164	143·5	61·4	77·1	50·0
166	86·9	163·0	102·1	48·5
168	60·6	119·1	59·2	7·1

TABLE IV.—Showing the First Four Harmonics of the Period between 1703 and 1755.

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
81	271·2	213·8	106·4	103·8
82	476·3	154·5	85·5	95·7
83	390·2	157·5	181·5	87·3
84	61·0	219·0	152·6	30·6
85	266·6	225·2	124·8	67·5
86	302·8	241·7	127·9	68·2
87	178·2	253·0	134·2	65·5
88	57·7	258·5	148·8	46·0
89	160·6	210·5	173·4	141·4
90	263·4	170·8	86·5	87·3
91	308·8	142·1	44·1	52·7
92	281·1	314·0	213·6	40·3
93	307·0	260·7	224·5	96·2
94	467·5	121·6	171·4	181·1
95	572·8	321·5	165·4	118·6
96	543·8	227·0	67·6	63·4
97	346·3	88·5	28·4	90·5
98	167·3	184·0	159·6	72·4
99	202·6	39·6	85·0	87·4
100	311·1	200·6	299·7	130·6
101	332·5	135·6	146·3	72·7
102	296·9	352·5	199·1	67·8
103	279·7	408·8	79·1	107·4
104	322·1	268·9	96·5	188·7
105	352·6	241·0	80·7	168·1
106	320·1	154·2	140·6	139·8
107	258·3	63·8	158·9	116·5
108	236·3	194·9	170·9	87·9
109	346·1	304·6	316·5	75·1
110	456·1	75·9	308·6	203·7
111	501·2	247·0	68·5	119·7
112	527·4	374·7	151·9	153·8
113	419·9	225·5	85·4	95·7
114	251·6	246·8	139·1	40·2
115	72·8	412·0	39·4	170·4
116	151·5	380·4	254·6	132·6
117	273·4	386·5	214·5	205·0
118	349·3	478·9	211·7	79·8
119	366·4	417·4	330·6	177·8

TABLE IV—*continued.*

Weeks.	1st harmonic.	2nd harmonic.	3rd harmonic.	4th harmonic.
120	342·6	202·8	127·2	76·2
122	291·0	237·0	120·0	144·8
124	337·9	216·4	208·2	215·3
126	431·3	98·0	207·5	140·8
128	405·2	104·5	131·2	64·2
130	287·7	201·7	306·9	29·4
132	342·1	258·5	246·2	85·1
134	535·2	214·2	77·0	234·1
136	575·0	428·2	192·4	193·8
138	400·3	180·4	306·4	111·9
140	194·8	268·8	178·6	64·6
142	365·8	178·5	284·4	129·9
144	553·6	196·0	210·0	171·1
146	594·1	152·3	143·8	383·6
148	484·6	88·9	76·7	43·8
150	390·0	282·5	169·3	29·5
152	386·8	425·6	192·3	117·4
154	441·6	322·9	412·6	181·4
156	463·4	356·4	253·1	200·1
158	456·0	561·9	214·5	302·4
160	441·0	415·6	84·8	133·6